Conditionally max-stable random fields

Martin Dirrler, University of Mannheim

Statistical inference of spatial extremal events is often based on the annual maxima method, where data are considered as lying in the domain of attraction of a max-stable process. It is well-known that such processes can be represented as the maxima of points of suitable Poisson point processes. We introduce a new class of models, which is based on Cox processes instead of Poisson processes, and show that the new models lie in the (functional) max-domain of attraction of familiar models. Therefore, they have the potential to explain spatial extreme observations on a smaller time scale whence the annual maxima can still be approximated by known max-stable models. Further assumptions on the Cox process are specified in order to make inference feasible. We propose a multi-stage procedure to estimate the parameters of the underlying Cox process. Non-parametric estimation of the intensity function forms an intermediate step. The performances of these procedures are assessed in a simulation study.

A Latent Mixture Model for Extreme Winds

Gregory Bopp, Pennsylvania State University

Understanding dependence among extreme weather events plays an important role in adequately designing infrastructure to withstand anticipated stresses. Models that allow for either asymptotic dependence or independence while retaining marginal components that belong to an extremal family of distributions are still under active development. I will present a Bayesian version of the hierarchical latent process model for temporal extremes developed by Bortot and Gaetan in 2013. Exceedances over a high threshold are modeled through a gamma mixture of exponential distributions in two stages. An observation model of conditionally independent exceedances given a latent stage are modeled according to an exponential distribution. The latent stage is taken to be a first order Markov process with Gamma margins. Together these yield dependent exceedances that marginally follow a generalized Pareto distribution. This model is applied to a time series of Santa Ana winds, which are notorious for knocking over utility poles and igniting brush fires.

Inference for Archimax Copulas

Simon Chatelain, Université Lyon 1 and McGill University

Archimax copulas, which generalize both extreme-value and Archimedean families, were proposed in the bivariate case by Capéraà, Fougeres, and Genest (2000) and extended to arbitrary dimensions by Charpentier, Fougeres, Genest and Neslehova (2014). Interesting properties of this family include a stochastic representation, known maximum/minimum domains of attraction as well as flexibility in terms of tail dependence and asymmetry. Therefore, this family could offer a new approach for modeling joint extremes. Under certain regularity assumptions, we are able to show weak convergence of the empirical copula process with respect to weighted metrics using the main result from Berghaus, Bücher and Volgushev (2016). This allows us to establish convergence of estimators for the Pickands dependence function. The use of self-nested diagonals or Kendall’s $\tau$ leads to a joint estimation procedure.
A Poisson process reparameterisation for Bayesian inference for extremes
Paul Sharkey, Lancaster University
A common approach to modelling extreme values is to consider the excesses above a high threshold as realisations of a non-homogeneous Poisson process. While this method offers the advantage of modelling using threshold-invariant extreme value parameters, the dependence between these parameters makes estimation more difficult. We present a novel approach for Bayesian estimation of the Poisson process model parameters by reparameterising in terms of a tuning parameter $m$. This talk will present a method for choosing the optimal value of $m$ that near-orthogonalises the parameters, which is achieved by minimising the correlation between the asymptotic posterior distribution of the parameters. This choice of $m$ ensures more rapid convergence and efficient sampling from the joint posterior distribution using Markov Chain Monte Carlo methods. Samples from the parameterisation of interest are then obtained by a simple transform. Results are presented in the cases of identically and non-identically distributed models for extreme rainfall in Cumbria, UK.

Modeling power laws in directed social networks
Phyllis Wan, Columbia University
We consider the directed preferential attachment network growth model. Samorodnitsky et al. (2016) have shown that the joint distribution of the in- and out-degree has jointly regularly varying tails. In the case where only one snapshot of the network is observed, full MLE of the model is not possible. Inference using tail empirical estimation of the degree distributions, which has been a common practice in the literatures, performs poorly in learning the model. In this talk, we present a superior estimation method based on pseudo-likelihood that gives consistent estimates for the model parameters.

Determining the dependence structure of multivariate extremes
Emma Simpson, Lancaster University
Asymptotic independence and asymptotic dependence are important considerations when modelling multivariate extremes. There may be a complicated structure to this asymptotic behaviour, with only certain subsets of variables taking their largest or smallest values simultaneously, and this structure should be exploited when developing and fitting models for multivariate extremes. In this talk, I will discuss our current work on determining the dependence structure of multivariate extremes. We consider variables in terms of their radial and angular components: a common technique in extreme value theory. In this case, the angular components lie in a unit simplex and, conditioning on the radial components being large, mass on the various sub-simplices corresponds to different subsets of variables being asymptotically dependent. Our current approach involves dividing the simplex into sections, and finding the conditional probability that a point lies in each section given that it is extreme in terms of the radial component. I will discuss the benefits of this approach, and potential areas in which the method could be improved.
A SIMULTANEOUS AUTOREGRESSIVE MODEL FOR SPATIAL EXTREMES
Miranda Fix, Colorado State University

Areal data is an important subclass of spatial data, e.g. public health data at the county level or gridded climate model output. However, current models for spatial extremes which characterize spatial tail dependence, such as max-stable models, are geostatistical in nature and have proven to be difficult to fit to spatial datasets with many locations. In classical spatial statistics, the simultaneous autoregressive (SAR) model for areal data constructs a simple spatial model which captures spatial dependence given a neighborhood structure. We apply recent results on transformed linear operations for regularly varying random vectors with tail index $\alpha = 2$ (Cooley and Thibaud, 2016) to develop an analogous SAR model for extremes. We will describe the model and how to simulate from it, the resulting tail pairwise dependence matrix which depends on the neighborhood structure and autoregressive parameter, and discuss preliminary methods for estimation and inference.

A PARAMETRIC VARIOGRAM MODEL BRIDGING BETWEEN MIXING AND NON-ERGODIC BROWN-RESNICK PROCESSES
Olga Moreva, University of Mannheim

We present a simple isotropic variogram model with two parameters that bridges between the class of bounded variograms and the class of unbounded variograms. The model includes the power variogram for the fractional Brownian motion, a modified De Wijsian model, the generalized Cauchy model and the multiquadric model. One parameter of the model controls the smoothness of the process. The other parameter controls the long range behaviour and allows for a smooth transition between stationary and intrinsically stationary second order processes in a Gaussian framework, i.e., between mixing and non-ergodic Brown-Resnick processes. Hence, our parametric model offers a simple statistical inference, particularly when the kind of causative process for the spatial extremes is not clear. In the talk, we will also present approaches for the simulation of corresponding Gaussian random fields.

MAX K-ARMED BANDITS: ON THE EXTREMEHUNTER ALGORITHM AND AN ALTERNATIVE APPROACH
Mastane Achab, Télécom ParisTech

We address the max $K$-armed bandit problem, which consists in sequentially allocating resources so as to detect extreme values. We improve the analysis of the ExtremeHunter algorithm from Carpentier and Valko (2014), proving that our analysis is tight. Moreover, we propose an alternative approach to the Extreme Bandits based on a reduction to a more classical bandit problem. We compare the two approaches in some numerical experiments.

AN INVARIANCE PRINCIPLE FOR SUMS AND RECORD TIMES FOR REGULARLY VARYING STATIONARY SEQUENCES
Hrvoje Planinić, University of Zagreb

It is known that point process convergence results are a useful tool in the analysis of extremal properties of dependent heavy-tailed data, where clustering of large observations usually occurs. Unfortunately, as observed by Hult and Samorodnitsky (2010), due to time scaling, existing point process convergence results typically lose the information about the order at which large observations occur within the cluster. However, this order is extremely important when, for example, studying ruin probabilities or the asymptotic behaviour of record times. We present a new type of point process convergence result for weakly dependent stationary and regularly varying sequences which preserves this kind of information.
We further apply our method to analyze record times in a sequence of dependent stationary observations, even when their marginal distribution is not necessarily regularly varying. Under certain restrictions on dependence between the observations, we show that the record times, after scaling, converge to a relatively simple compound scale invariant Poisson process.

Furthermore, by going beyond the existing asymptotic theory, we are able to prove a new functional limit theorem for partial sums for a wide class of applied time series models, for which standard limiting theory in the space $D$ of cadlag functions does not apply.

This presentation is based on the joint work with Bojan Basrak and Philippe Soulier.
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