WORKSHOP ON
Risk Quantification and Extreme Values in Applications

FEBRUARY 15–17, 2017

ABSTRACTS

Program committee: Valérie Chavez-Demoulin
Anthony C. Davison
Sebastian Engelke

Organizer: Sebastian Engelke
Wednesday, Feb. 15th

9:00 – 10:20

**An extreme value approach for modeling Operational Risk losses depending on covariates**

**Paul Embrechts, ETH Zürich**

In financial risk management, Operational Risk data typically appear as entries in a $BL \times RT$-matrix where $BL$ stands for the number of business lines, and $RT$ corresponds to risk types. For instance ($BL$) Corporate Finance and ($RT$) Internal Fraud. Banks and insurance companies often, at least for internal purposes, model Operational Risk losses based on such a data matrix and use a particular risk measure to be statistically estimated. From a mathematical point of view the (internal) data available consists of $BL \times RT$ marked point processes. A typical example consists of a ($BL = 8$, $RT = 7$)-matrix, with historical data in each cell. As risk measure one often takes a high quantile (99.9%) of the total matrix loss distribution function over a one year horizon (referred to in the industry as a one-year Value-at-Risk). In order to analyze this problem we introduce a dynamic version of Extreme Value Theory (EVT) introducing as co-variables rows, columns from the data matrix as well as time. Currently, the use of the so-called Advanced Measurement Approach (AMA) and the Loss Distribution Approach (LDA) for risk capital purposes for Operational Risk under the Basel III guidelines are hotly disputed. I will comment on the current discussion and frame our work accordingly. This talk is based on joint work with Valérie Chavez-Demoulin (UNIL) and Marius Hofert (University of Waterloo).

**Analysing model risk for probabilities of default**

**Dirk Tasche, Swiss Financial Market Supervisory Authority FINMA**

How to forecast this year’s portfolio-wide credit default rate based on last year’s default observations and the current credit rating distribution? It is common to assume that this year’s grade-level default rates will be the same as last year. The grade-frequency weighted average of the default rates then is taken as an estimate of this year’s portfolio-wide default rate. In the machine learning community, the assumption underlying this approach is called ‘covariate shift’. Typically, this way not much change of the default rate is predicted — which suggests that model risk is high in this case. We analyse the model risk by comparing the covariate shift estimation method to other methods, based on different assumptions of the portfolio risk evolution. We show that the default rate change predicted by the covariate shift method is always smaller than the change predicted by assuming that the ratios of the grade frequencies of defaulters and non-defaulter remain constant. This observation suggests that the maximum of these two estimates can used to mitigate the model risk.

10:50 – 12:30

**Robust bounds in multivariate extremes**

**Jevgenijs Ivanovs, Aarhus University**

In this work we provide asymptotic bounds on exceedance probabilities that are robust against misspecification of the extremal dependence model. They arise as the optimal values when optimizing the statistic of interest over all dependence models within some neighbourhood of the reference model. A certain relaxation of these bounds results in surprisingly simple and explicit expressions, which we propose to use in applications. We show the effectiveness of the robust approach compared to classical confidence bounds when the model is misspecified.
Higher order elicitability: Strictly consistent scoring functions for (Value at Risk, Expected Shortfall) and beyond

Tobias Fissler, University of Bern

A statistical functional, such as the mean or the median, is called elicitable if there is a scoring function or loss function such that the correct forecast of the functional is the unique minimizer of the expected score. Such scoring functions are called strictly consistent for the functional. The elicitability of a functional opens the possibility to compare competing forecasts and to rank them in terms of their realized scores.

We explore the notion of higher order elicitability, that is, we investigate the question of elicitability for higher-dimensional functionals. As a result of particular applied interest we show that the pair (Value at Risk, Expected Shortfall) ((VaR, ES)) is elicitable despite the fact that ES itself is not. More generally, we give a characterization of the class of strictly consistent scoring functions for this pair, making use of a higher dimensional version of Osband’s principle. The elicitability of the pair (VaR, ES) leads the way to comparative backtests of Diebold-Mariano type.

Moreover, we discuss the choice of a ‘good’ scoring function. While strict consistency is commonly undoubted to be a minimal requirement of a ‘good’ scoring function, we introduce further appealing properties that should be satisfied, such as homogeneity, convexity or order-sensitivity of scoring functions. These results should give guidance in the choice of a scoring function in general, and in particular for the pair (Value at Risk, Expected Shortfall).

References:

How is elicitability relevant for backtesting?

Johanna F. Ziegel, University of Bern

Independently, Weber (2006) and Gneiting (2011) have shown that Expected Shortfall (ES) is not elicitable in contrast to Value at Risk (VaR). Roughly, elicitability of a risk measure means that it can be obtained as the minimizer of an expected loss function. This negative result continues to hold for all spectral risk measures (except for the mean) and the only coherent risk measures that are elicitable are certain expectiles. However, we were able to show recently that ES is jointly elicitable with VaR, and, more generally, a large class of spectral risk measures is elicitable of higher order (Fissler and Ziegel, 2016). There is little debate that elicitability is a useful property for model selection, estimation, generalized regression, forecast comparison, and forecast ranking. But the non-elicitability of ES has lead to a lively debate about the relevance of elicitability for backtesting. Contributing to this discussion, we would like to clarify that elicitability is not important for the traditional approach to backtesting. However, we argue that elicitability is valuable for comparative backtesting (Fissler et al., 2016, Nolde and Ziegel, 2015). We illustrate the proposed approach for VaR and ES jointly and for VaR alone. Joint work with: Tobias Fissler and Tilmann Gneiting.
References:

A continuous updating weighted least squares estimator of tail dependence in high dimensions

**Anna Kiriliouk, Université catholique de Louvain**

Likelihood-based procedures are a common way to estimate tail dependence parameters. They are not applicable, however, in non-differentiable models such as those arising from recent max-linear structural equation models. Moreover, they can be hard to compute in higher dimensions. An adaptive weighted least-squares procedure matching nonparametric estimates of the stable tail dependence function with the corresponding values of a parametrically specified proposal yields a novel minimum-distance estimator. The estimator is easy to calculate and applies to a wide range of sampling schemes and tail dependence models. In large samples, it is asymptotically normal with an explicit and estimable covariance matrix. The minimum distance obtained forms the basis of a goodness-of-fit statistic whose asymptotic distribution is chi-square. Extensive Monte Carlo simulations confirm the excellent finite-sample performance of the estimator and demonstrate that it is a strong competitor to currently available methods. The estimator is then applied to disentangle sources of tail dependence in European stock markets.
14:00 – 15:20

Non Linear Models for Dependence Structures
Valérie Chavez-Demoulin, Université de Lausanne

In this talk, we present two methodologies addressing the modeling of dependence structures taking into account the effect of predictors. The two methodologies rely on the flexible generalized additive modeling framework. In the first part, we consider dependence or concordance measures that are solely functions of the copula and not on the margins: rank correlation coefficients or tail-dependence coefficients represent natural choices. We propose a maximum penalized log-likelihood estimator and derive its $\sqrt{n}$-consistency and asymptotic normality. This is a joint work with Thibault Vatter. In the second part, we consider the dependence structure of max-stable random vectors characterized by its Pickands dependence function. We develop a new flexible semiparametric method for the estimation of nonstationary multivariate Pickands dependence functions. The proposed construction is based on an accurate max-projection allowing to pass from the multivariate to the univariate setting and to rely on the generalized additive modelling framework. The resulting estimator of the Pickands function is regularized, in the bivariate case, using constrained median smoothing B-splines and bootstrap confidence intervals are constructed. This is a joint work with Linda Mhalla and Philippe Naveau. We present the results from simulation studies and apply the new methodologies to high frequency financial data.

Why risk is so hard to measure
Chen Zhou, Erasmus University Rotterdam and De Nederlandsche Bank

This paper analyzes the robustness of standard techniques for risk analysis, with a special emphasis on the Basel III risk measures. We focus on the difference between value-at-risk and expected shortfall, their small sample properties, the scope for manipulating risk measures and how estimation can be improved. Overall, the paper find that risk forecasts are extremely uncertain at low sample sizes, with value-at-risk more accurate than expected shortfall, while value-at-risk is easily manipulated without violating regulations. Finally the implications for practitioners and regulators are discussed along with best practice suggestions.

15:50 – 16:30

Generalized Additive Modeling of Nonstationary Multivariate Extremes
Linda Mhalla, Université de Genève

In this talk I will introduce smooth models for nonstationary extremal dependence structures. Our approach is constructed from covariate-dependent versions of well-known parametric models for the spectral density of a multivariate extreme value distribution, such as the logistic, Dirichlet, and the Hüsler–Reiss models. Fitting is conducted by a maximum penalized likelihood estimator based on the Poisson Point Process approach. The methods are applied to simulated data, and a case study in Finance is used to motivate the need for theory and methods, as well as to illustrate the main concepts introduced along the talk. This is joint work with Valérie Chavez-Demoulin and Miguel de Carvalho.
AMBIGUITY SETS FOR EXTREME EVENT DISTRIBUTIONS
Georg Pflug, University of Vienna

Stochastic optimization problems (e.g. the design of insurance contracts) are usually formulated on the basis of a known distribution for the uncertainties. In distributionally robust stochastic optimization, we assume only that the true model lies in a set of models compatible with our observations and we solve a minimax problem. The question is how to describe the set of models if extreme events are crucial for the problem. Usual Wasserstein balls seem to be not quite appropriate. One way is to use Wasserstein balls with together with a distorted distance on the real line.
Thursday, Feb. 16th

9:00 – 10:20

**Algorithms to simulate max-stable random fields**

**Martin Schlather, Universität Mannheim**

In the talk I give an overview over existing simulation algorithms for the simulation of max-stable random fields. In more details, the simulation algorithm based on normalized spectral representation will be considered, which is based on nested Markov Chains.

**Feature clustering for extreme events analysis, with application to extreme stream-flow data**

**Anne Sabourin, Université Paris-Saclay**

In a high dimensional context ($d > 50$), a natural first step when analyzing the dependence structure of extreme event is dimension reduction. Earlier works have defined sparsity in multivariate extremes as the concentration of the angular measure on low dimensional subspheres, and have proposed an algorithm detecting such a pattern in cases where the latter is apparent. Given a dataset that exhibits no clear sparsity pattern we propose an alternative summary of the dependence structure allowing to cluster together the features that are ‘dependent at extreme level’, i.e. that are likely to take extreme values simultaneously. To bypass the computational issues that arise when it comes to dealing with possibly $O(2^d)$ subsets of features, our CLEF algorithm (CLustering Extreme Features) exploits the graphical structure stemming from the definition of the clusters, similarly to the Apriori algorithm, which reduces drastically the number of subsets to be screened. Results on simulated and real data show that our method allows a fast recovery of a meaningful summary of the dependence structure of extremes. In terms of asymptotic guarantees, a re-writing of the dependence criterion involved in CLEF shows that a set is declared ‘empty’ if a certain linear combination of extremal coefficients is below/above a threshold. This suggests an interpretation of CLEF as a sequence of statistical tests and opens the road to asymptotic statistical guarantees concerning the output.

10:50 – 12:30

**Asymptotic properties of likelihood estimators in multivariate extremes**

**Clément Dombry, Université de Franche-Comté**

Max-stable distributions and processes are important models for extreme events and the assessment of tail risks. The full, multivariate likelihood of a parametric max-stable distribution is complicated and only recent advances enable its use in both frequentist and Bayesian statistics. The asymptotic properties of the maximum likelihood estimator and the median of the posterior distribution in multivariate extremes are mostly unknown. We provide natural conditions on the exponent function and the angular measure of the max-stable distribution that ensure asymptotic normality of these likelihood estimators. We show the effectiveness of this result by applying it to popular parametric models in multivariate extreme value statistics. and to the most commonly used spatial max-stable processes.
Estimation of max-linear models on directed acyclic graphs

Nadine Gissibl, Technische Universität München (TUM)

We consider recursive structural equation models where all random variables can be written as a max-linear function of their parents and independent noise variables. We develop estimation methods for this model class. Firstly, we assume that the structure of the corresponding directed acyclic graph is given. We derive a generalized maximum likelihood estimator for the weights of the max-linear structural equation model. Secondly, we address the important question of identifiability and estimation of its causal structure. We propose a statistical method and an algorithm based on our theoretical findings. This talk is based on joint work with Claudia Klüppelberg and Steffen Lauritzen.

High-dimensional peaks-over-threshold inference with generalized risk functionals

Raphaël de Fondeville, EPFL

Classical spatial models for extremes rely on block maxima, but this approach is limited by computational considerations to a few dozen variables (Wadsworth and Tawn, 2013; Huser and Davison, 2014). In order to get a better understanding of extremal dependence and reduce model uncertainties, exploitation of gridded datasets, such as climate models, is necessary. Generalized Pareto processes (Dombry and Ribatet, 2015), based on a peaks-over-threshold approach, use single extreme events, generalize the notion of exceedance, and have simpler mathematical expressions.

For spatial modelling, we focus on the Brown–Resnick model, which relies on classical Gaussian models widely used in applications. An efficient algorithm for censored likelihood allows us to perform inference with hundreds of locations. For higher dimensions and generalized risk functionals, we develop an estimator based on the gradient score (Hyvärinen, 2005) with a complexity similar to likelihood-based inference methods for Gaussian field.

We apply our method to fit a model for extreme rainfall over Florida on a grid with 3600 locations for two types of exceedances: locally intense and areal cumulated rainfall. We can then use the model to generate new extreme events with unobserved intensity and spatial pattern. This is joint work with Prof. Anthony Davison. References:

**Local estimation of the conditional stable tail dependence function**  
**Mikael Escobar-Bach, Syddansk Universitet**  
In this talk, we consider the local estimation of the stable tail dependence function when a random covariate is observed together with the variables of main interest. Our estimator is a weighted version of the empirical estimator adapted to the covariate framework, assuming we have data available from a distribution in the max-domain of attraction of a multivariate extreme value distribution. We provide the main asymptotic properties of our estimator, when properly normalized, in particular the convergence of the empirical process towards a tight centered Gaussian process. The finite sample performance of our estimator is illustrated on a small simulation study.

**14:00 – 15:20**  
**Modelling spatial processes with unknown extremal dependence**  
**Jenny Wadsworth, Lancaster University**  
Many environmental processes exhibit weakening spatial dependence as events become more extreme. Well-known limiting models, such as max-stable or generalized Pareto processes, cannot capture this, which can lead to a preference for models that exhibit a property known as asymptotic independence. However, weakening dependence does not automatically imply asymptotic independence, and whether the process is truly asymptotically (in)dependent is usually far from clear. The distinction is key as it can have a large impact upon extrapolation, i.e., the estimated probabilities of events more extreme than those observed. We present a single spatial model that is able to capture both dependence classes in a parsimonious manner, and with a smooth transition between the two cases. The model covers a wide range of possibilities from asymptotic independence through to complete dependence, and permits weakening dependence of extremes even under asymptotic dependence. Censored likelihood-based inference for the implied copula is feasible in moderate dimensions due to closed-form margins. Time permitting, an application of the model to a dataset of hindcast significant wave heights will be discussed.

**Modelling Extremes with Random Forests**  
**Stefan Wager, Stanford University**  
Forest-based methods are being used in an increasing variety of statistical tasks, including causal inference, survival analysis, and quantile regression. Extending forest-based methods to these new statistical settings requires specifying tree-growing algorithms that are targeted to the task at hand, and the ad-hoc design of such algorithms can require considerable effort. In this talk, I will present a unified framework for the design of fast forest-based procedures for tasks that can be characterized by heterogeneous estimating equations. The method, gradient forest, starts by viewing forests as providing a type of data-adaptive neighborhood function that can be used for local estimation; the key challenge is then in the computationally efficient construction of neighborhood functions that can meaningfully express heterogeneity in the quantity of interest. To do so, we grow trees by recursively applying a pre-processing step where we label each observation with gradient-based pseudo-outcomes, followed by a regression step that runs a standard CART regression split on these pseudo-outcomes. Then, given this abstract framework, I will discuss applications of gradient forests to quantile estimation and to modelling extremes.
Friday, Feb. 17th

9:00 – 10:20

**Downscaling of spatial extremes**

**Marco Oesting, Universität Siegen**

Meteorological gridded datasets, such as climate model output, exhibit different statistical behavior than measurements at gauging stations. A possible explanation is that these products result from some form of aggregation, for instance averages or maxima over a geographical region. In order to use this data for accurate risk assessment, downscaling to point locations is required. Methods have been developed for averages, but there is only little work for extremes. In this paper, we derive the joint tail behavior of general aggregation functionals for data in the domain of attraction of a max-stable process. Particularly simple, explicit formulas arise for the important case of Brown-Resnick processes. We further show that these asymptotic results provide the bases for statistical downscaling of extremes.

**Bridging Asymptotic Independence and Dependence in Spatial Extremes using Gaussian Scale Mixtures**

**Raphaël Huser, King Abdullah University of Science and Technology (KAUST)**

Gaussian scale mixtures are constructed as Gaussian processes with a random variance. They have non-Gaussian marginals and can exhibit asymptotic dependence unlike Gaussian processes, which are always asymptotically independent except for perfect dependence. Motivated by the analysis of spatial extremes, we propose a flexible but parsimonious Gaussian scale mixture copula model, which smoothly interpolates from asymptotic dependence to independence. We show how this new model can be fitted to high threshold exceedances using a censored likelihood approach, and we demonstrate that it provides valuable information about tail characteristics. The methodology will then be illustrated with an application to wind speed data in the Pacific Northwest, US, showing that it adequately captures the data’s extremal properties.

10:50 – 12:30

**Likelihood based change point detection in samples of independent block maxima with adjustment for multiple testing under dependency**

**Sven Buhl, Technische Universität München (TUM)**

Given a sample of independent random variables $X_1, \ldots, X_n$, we use likelihood ratio statistics to test simultaneously at each single sample point the hypothesis of a change in the parameters of their distribution. Our leading example consists of independent GEV distributed block maxima. The corresponding $p$-values need to be adjusted due to multiple testing. For this purpose, we control the false discovery rate (FDR) using the Benjamini-Hochberg (BH) procedure proposed in [1]. In case of dependence between the $p$-values, [2] proved that the BH procedure keeps controlling the FDR if the test statistics satisfy certain positive dependency properties. We verify that the asymptotic distribution of the vector of likelihood ratio statistics is multivariate totally positive of order 2 (MTP$_2$, cf. [3]), which in turn implies the required properties. We investigate the finite sample behaviour of the proposed test procedure in a simulation study and apply it to real environmental data sets. Finally, we compare the results with tests based on probability weighted moments recently proposed in [4]. This talk is based on a joint work with Anthony Davison.
References:


**Modelling Extremes of Markov Chains**

**Thomas Lugrin, EPFL and Lancaster University**

Standard approaches to modelling extremes of stationary time series with short-range dependence typically involve pre-processing of the series and marginal modelling of subjectively selected peaks. Such methods heavily rely on a pre-processing stage, which can yield badly biased estimates of related risk measures. Assuming that the series is stationary and Markovian, we use an all-in-one approach where the marginal distribution is fitted simultaneously with a conditional tail model for the Markov kernel. The dependence model covers a broad class of extremal dependence structures and includes information from non-extreme events which contribute through a censored likelihood. The model provides a unified framework in which marginal and dependence features can be estimated simultaneously, yielding efficient estimates and providing a natural assessment of the full uncertainty of the model. An application to river flows illustrates our approach.

**Estimation of Extreme Sea Levels using a Spatial Extreme Value model**

**Jonathan Jalbert, McGill University**

All over the world, floods are considered to be the most catastrophic of natural disasters, both in terms of damages and the number of victims. In Canada, insurance companies are beginning to offer protection against these events. In particular, they have products for flooding caused by intense rainfall as well as river floods. However, currently, no protection exists for coastal floods. The goal of this project is to estimate extreme water levels and the subsequent risk of coastal flooding in 3 regions of Canada: the Atlantic, Pacific and the Great Lakes. To accomplish this, a spatial model for extreme values is used to model the annual maximum sea levels as measured by buoys in the regions of interest. Additionally, with this model, one will be able to interpolate the results to locations with no observations. In doing so, the creation of coastal flooding maps indicating the potential risk of disaster is possible as well as the ability to estimate the potential damages incurred so as define the appropriate premium for coastal flood protection.
Two decompositions of dependence for multivariate extremes

Emeric Thibaud, EPFL

Multivariate regular variation is a framework which is useful for describing tail dependence and estimating probabilities of multivariate extreme events. Dependence for regularly-varying random vectors is described by the angular measure. In large dimensions, this measure is difficult to estimate. Inspired by principal component analysis (PCA) in the non-extreme setting, we propose two decompositions of a matrix which summarizes pairwise tail dependence in a regularly-varying random vector. The first decomposition is useful to understand the largest modes of dependence as is done with traditional PCA. The second decomposition is useful for calculating probabilities of extreme regions and for simulation. We illustrate methods with two applications: daily precipitation measurements at 44 stations in Switzerland, and daily return data from 30 financial categories. This is joint work with Dan Cooley (Colorado State University).

13:30 – 14:50

Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection

Philippe Naveau, Laboratoire des Sciences du Climat et l’Environnement, CNRS

In statistics, extreme events are often defined as excesses above a given large threshold. This definition allows hydrologists and flood planners to apply Extreme-Value Theory (EVT) to their time series of interest. Even in the stationary univariate context, this approach has at least two main drawbacks. First, working with excesses implies that a lot of observations (those below the chosen threshold) are completely disregarded. The range of precipitation is artificially chopped down into two pieces, namely large intensities and the rest, which necessarily imposes different statistical models for each piece. Second, this strategy raises a nontrivial and very practical difficulty: how to choose the optimal threshold which correctly discriminates between low and heavy rainfall intensities. To address these issues, we propose a statistical model in which EVT results apply not only to heavy, but also to low precipitation amounts. Our model is in compliance with EVT on both ends of the spectrum and allows a smooth transition between the two tails, while keeping a low number of parameters. This work is based on a recent article (Naveau, Huser, Ribereau, Hannart, WRR, 2016), and on I will also discuss current extensions in a non-parametric framework (see the PhD work of P. Tencaliec) and extensions to the multivariate context (joint work with R. Huser and A. Hannart).

G-WEX: a multi-site generator for extreme weather simulation

Anne-Catherine Favre, Institut National Polytechnique de Grenoble

Many multi-site stochastic models have been proposed for the generation of daily precipitation, but they generally focus on the reproduction of the statistical properties of low to high precipitation events. In this work, we develop a multi-site model targeting the reproduction of extreme events at different temporal and spatial scales. This precipitation model, named G-Wex, relies on the structure proposed by Wilks (1998), for which the process representing the precipitation occurrences at the different stations is independent to the process generating the amounts of the precipitation events.
Different improvements have been brought to the version proposed by Wilks (1998) in order to have a better reproduction of extreme precipitation events. The statistical distribution model (heavy tailed) of precipitation intensity originally proposed by Papastathopoulos and Tawn (2013) and investigated in more details in Naveau et al. (2016) fits adequately low, moderate and very heavy precipitation intensities. This distribution, in combination with a method of regionalization, provides a reliable and robust adjustment of the most extreme precipitation intensities at each station. Temporal and spatial dependence of precipitation intensities is handled using a Multivariate Autoregressive Model of order 1 — \( \text{MAR}(1) \).

Different G-Wex model variations are applied to a dense network of gauges of the Aare-Rhine hydrological system in Switzerland. Performances are then evaluated at different temporal (e.g. 1-day and 3-day annual maxima) and spatial scales (e.g. at the stations and for different catchment sub-divisions).
Poster session, Wednesday, Feb. 15th

17:30 – 19:30

MAX K-ARMED BANDITS: ON THE EXTREMEHUNTER ALGORITHM AND AN ALTERNATIVE APPROACH
Mastane Achab, Télécom ParisTech

We address the max K-armed bandit problem, which consists in sequentially allocating resources so as to detect extreme values. We improve the analysis of the ExtremeHunter algorithm from Carpentier and Valko (2014), proving that our analysis is tight. Moreover, we propose an alternative approach to the Extreme Bandits based on a reduction to a more classical bandit problem. We compare the two approaches in some numerical experiments.

STATISTICAL REGIONALIZATION FOR ESTIMATION OF EXTREME RIVER DISCHARGES
Peiman Asadi, EPFL

The accurate quantification of peak flow values with long return periods is crucial for national agencies in order to design effective flood protection and reduce economic and ecological costs. For gauging stations on a river network with long discharge records univariate extreme value theory provides reliable tools for model fitting, identification and assessment of parameter uncertainty. At ungauged locations, however, where no observations are available, these methods are no longer applicable. We propose a statistical regionalization approach that identifies the optimal region with gauged stations that are hydrologically similar to the target ungauged location. A regression model is then used to transfer information on extreme discharges from this region of influence to the ungauged site. This estimation procedure is applied to discharge data on Swiss river networks and is compared to competing methods.

STATISTICAL MODELS OF WIDESPREAD FLOOD EVENTS AS A CONSEQUENCE OF EXTREME RAINFALL AND RIVER FLOW
Anna Barlow, Lancaster University

We consider the modelling of extreme river flow and estimation of the tail of the loss distribution for a portfolio given these extreme values. In this poster two main areas are discussed in particular. Firstly, the point at which we decide to analyse data often coincides with some extreme event and so estimation of the extreme value model parameters based on such a sample may be biased. We call this the ‘stopping bias’ and consider incorporating various stopping rules into the generalised extreme value likelihood to account for this bias.

Secondly, in current practice, estimation of the tail of the loss distribution for a portfolio is done by simulating and summing the losses for a large number of events and risks. This is often very computationally expensive so we instead propose to reduce the number of simulations necessary by considering bounds on the loss per risk derived from concentration inequalities.
Insurance premium under high ambiguity with application in extreme climatic events

Corina Birghila, University of Vienna

The insurance industry depends on probabilistic calculations of risk. Especially in the case of extreme events, high ambiguity concerning the occurrence and the magnitude of losses increases the difficulty of managing and estimating risk. In order to incorporate robustness into the insurance portfolio, our approach includes the baseline model of losses, as well as all the models that are at a specified $\epsilon$-distance from the baseline model. These models can explain the data in a similar way. Hence, the family of alternative models constructed in this way compensate for the inherent ambiguity that is present in these natural phenomena. The ambiguity set depends heavily on the chosen metric. We propose a modified version of Wasserstein distance which penalizes more the differences between models at higher quantiles. Numerical results are used to compare the classical Wasserstein distance to the proposed distance, more sensitive to the tail of the extreme value distributions.

Similarity-Based Clustering for Stock Market Extremes

Miguel de Carvalho, University of Edinburgh

The analysis of the magnitude and dynamics of extreme losses in a stock market is essential from an investors viewpoint. We develop methods of similarity-based clustering for statistics of heteroscedastic extremes which allow us to assemble stocks that are more similar from the viewpoint of the scedasis function and/or tail index. Clustering is here performed in a product-space and a weighting parameter is used to control if more emphasis should be put on the scedasis function or on the tail index. Another contribution of our paper rests on the study of the way that the modes of several scedasis functions are spread over time. Our analysis reveals some interesting connections between the magnitude and dynamics of extreme losses on the London stock exchange with the corresponding economic sectors of stocks. Joint work with R. Rubio and R. Huser.

Extreme value analysis of electricity demand in the UK

Stephen Chan, The University of Manchester

For the first time, an extreme value analysis of electricity demand in the UK is provided. The analysis is based on the generalized Pareto distribution. Its parameters are allowed to vary linearly and sinusoidally with respect to time to capture patterns in the electricity demand data. The models are shown to give reasonable fits. Some useful predictions are given for the value at risk of the returns of electricity demand.

A reinforced-urn process modeling of recovery rates and loss given default

Dan Cheng, Delft University of Technology

We propose a new approach to recovery rates and LGD modeling, by developing a nonparametric survival model based on Pólya-reinforced urns. The model allows for the elicitation and exploitation of prior knowledge, if available, and for the constant update of this information over time, as soon as new data become available. Its probabilistic properties are studied in detail. We show how the model can be used to perform Bayesian prediction about the recovered amounts and the time of recovery. The performances of the model are tested via simulations.
Testing the maximal rank of the volatility process for continuous diffusions observed with noise

Tobias Fissler, University of Bern

We present a test for the maximal rank of the volatility process in continuous diffusion models observed with noise. Such models are typically applied in mathematical finance, where latent price processes are corrupted by microstructure noise at ultra high frequencies. Using high frequency observations, we construct a test statistic for the maximal rank of the time varying stochastic volatility process. Our methodology is based upon a combination of a matrix perturbation approach and pre-averaging. We show the asymptotic mixed normality of the test statistic and obtain a consistent testing procedure. We demonstrate the applicability of the results and their limitations with a simulation study showing the performances on finite samples.

References:

The Bauer Simplices of Stable Tail Dependence Functions and their extremal elements

Timo Fuller, Universität Würzburg

The concept of an Angular Set is introduced, which is a generalization of normed surfaces. An Angular Measure is a probability measure on an Angular Set. Angular Measures generate stable tail dependence functions the usual way and still the generator distributions are unique. Also convergence in distribution of the Angular Measure is equivalent to point-wise convergence of the stable tail dependence function. Generalizing a result of Ressel, the set of stable tail dependence functions created with an Angular Set is a Bauer simplex, which is a special type of convex, sequentially compact set. Its extremal elements correspond to max-factor models (as seen in works of Segers, Einmahl, Krajina, also by the name max-linear model in Stoev, Wang) with a single factor. Convex combinations of those extremal elements result in the max-factor models. Krein Milman theorem then implies every extreme value copula can be approximated by max-factor models.

Risk Analysis for Dzud in Mongolia

Masahiko Haraguchi, Columbia University

The objective of this study is to conduct risk analysis for mass livestock mortality during winters, known as dzud, in Mongolia utilizing climate variables and paleoclimate data. There are two important climatic variables to predict dzud: summer precipitation and winter temperature. In terms of summer precipitation, this study estimates return periods of extreme drought conditions using tree-ring reconstructed Palmer Drought Severity Index (PDSI) based on block maximum and threshold approaches using non-stationary extreme value theories. The study discovers that only looking at General Extreme Value distribution without a threshold, there is a non-stationarity trend in tree-ring reconstructed PDSI data. However, the threshold approach indicates that extreme events in reconstructed PDSI values are stationary. In terms of winter temperature, the study estimates return periods of extreme cold temperatures in Mongolia using instrumental temperature data in Siberia. Based on simulated data from Siberia data, we discover that in the Southwest of Mongolia, temperatures for 100 year events are rising over time; in Northwest and East clusters in the Mongolia, the trend is not clear.
Contingent convertible bonds with the default risk premium

Hyun Jin Jang, Ulsan National Institute of Science and Technology

Contingent convertible bonds (CoCos) are hybrid instruments which are characterized by both features, debt and equity. CoCos are automatically converted into equities or written down principal when a capital-ratio of an issuing bank falls below a contractual threshold. This paper studies new methodology for pricing CoCos with a capital-ratio trigger and develops how to quantify issuing bank’s default risk which can happen in a post-conversion period for pricing CoCos. We model an equity price follows a geometric Brownian motion and a risk-weighted asset level is a random variable which reveals only at time of conversion, but its distribution may be progressively estimated at issuance. Under the framework, we quantify post-conversion risk as possibility of banks’ default as the moment when a capital-ratio hits a hypothetical default threshold after conversion. Also, we formulate the default risk premium caused by post-conversion risk of equity-conversion CoCos. In numerical simulations, semi-analytic results are compared with those from a Monte Carlo method and sensitivity analysis of CoCos is conducted for risk management. The empirical tests show equity-conversion CoCo market prices have been reflecting the default risk premium.

Some results on joint record events

Amir Khorrami Chokami, Università Bocconi

When $d = 1$, the two definitions coincide. In the univariate case, several results on records are already known, e.g. the books by Galambos (1987) and Arnold, Balakrishnan, and Nagaraja (1998). For example, it is a well known fact that in the univariate case the events \{X_j \text{ record}\} and \{X_k \text{ record}\}, for indices $k > j$, $j = 1, 2, \ldots$, are independent. We provide additional asymptotic results on the conditional joint survival and distribution functions of two subsequent records. From the limiting conditional joint distribution function, we then derive some measure of dependence between records such as the correlation. Different to the univariate case, in the multivariate case ($d \geq 2$) much less is known on records. Some existing results are Zott (2016) and Dombry, Falk and Zott (2016), among other. We investigate the problem of establishing which results in univariate records do not carry over to the multivariate case. Hence, we show which are instead the findings obtained with simple and complete records. This is joint work with Michael Falk and Simone Padoan.

Numerical schemes for derivative pricing under a Markov modulated Lévy process

Younhee Lee, Chungnam National University

We consider that the underlying asset follows a Markov modulated jump-diffusion process. In order to evaluate European and American options with variable parameters, we deal with a linear system of partial integro-differential equations (PIDEs) and a linear complementarity problem (LCP) respectively. An explicit-implicit method to solve the PIDE and the LCP numerically is proposed and then it leads to the tridiagonal linear system at each state of the economy and at each time step. It is proved that the proposed method is stable and has the second-order accuracy in the time and spatial variables. A variety of simulations are performed to show the results of the analysis.
Semi-parametric inference for asymptotic independence of sample extremes
Simone Padoan, Stefano Rizzelli, Università Bocconi

Modelling dependence among extremes is an important topic in multivariate extreme value theory (MEVT). The so-called asymptotic independence framework (Ledford and Tawn, 1996, 1997) originates as a useful approach for modelling the joint upper (lower) tails when the classical joint upper (lower) tail dependence coefficient of a random vector, say $X = (X_1, \ldots, X_d)$, is null. In such a case we say that the components of $X$ are asymptotically independent. In classical MEVT, when variables are asymptotically independent, the limiting distribution of the component-wise maxima (or multivariate extreme-value distribution) is equal to a product of independent marginal distributions. In some applications, the convergence of the distribution of the component-wise maxima to a multivariate extreme-value distribution may not be fast enough to obtain a usable format. In this regard, new theoretical developments have been provided by Ramos and Ledford (2009, 2011), who derived a limiting distribution for component-wise maxima analogous to that of the MEVT, but specifically for asymptotically independent variables (see also Wadsworth and Tawn, 2013). On the basis of the recent work by Marcon et al. (2016), we propose an hypothesis test for checking if the dependence structure of a sample of maxima is compatible with asymptotic dependence or alternatively with asymptotic independence. Following the approach of Ramos and Ledford, we introduce a Pickands dependence function for the asymptotic independence framework, denoted by $A_{\eta}$, analogous to the standard Pickands in the MEVT and we study its properties. Then, we suggest a functional semi-parametric estimator of $A_{\eta}$, which combines the empirical margins and the generalized probability weighted moment (GPWM) estimator of $\eta$ introduced in Guillou et al. (2014). Finally, we study the main asymptotic properties of the estimator. This is joint work with Armelle Guillou.

An invariance principle for sums and record times for regularly varying stationary sequences
Hrvoje Planinić, University of Zagreb

It is known that point process convergence results are a useful tool in the analysis of extremal properties of dependent heavy-tailed data, where clustering of large observations usually occurs. Unfortunately, as observed by Hult and Samorodnitsky (2010), due to time scaling, existing point process convergence results typically lose the information about the order at which large observations occur within the cluster. However, this order is extremely important when, for example, studying ruin probabilities or the asymptotic behaviour of record times. We present a new type of point process convergence result for weakly dependent stationary and regularly varying sequences which preserves this kind of information.

We further apply our method to analyze record times in a sequence of dependent stationary observations, even when their marginal distribution is not necessarily regularly varying. Under certain restrictions on dependence between the observations, we show that the record times, after scaling, converge to a relatively simple compound scale invariant Poisson process. Furthermore, by going beyond the existing asymptotic theory, we are able to prove a new functional limit theorem for partial sums for a wide class of applied time series models, for which standard limiting theory in the space $D$ of cadlag functions does not apply.

This presentation is based on the joint work with Bojan Basrak and Philippe Soulier.
EPFL – Workshop on Risk Quantification February 15–17, 2017

EXPERT ELICITATION AND EXTREME EVENTS
María Elena Rivera Mancía, Banco de México
In this study, an analysis of extreme data is performed by using a Bayesian approach that combines a parametric form for the center and a Generalized Pareto Distribution (GPD) for the tail of the distribution. We show how to construct a prior distribution based on measures that experts are familiar with, including the Value-at-Risk (VaR) and the expected shortfall (ES). The purpose of this work is to facilitate prior elicitation and reproduce expert judgment faithfully. We also implement techniques for the combination of expert opinions and examine how their opinions may influence the posterior distribution and how to build a prior distribution based on expert judgment rather than using non-informative priors. While this issue has been addressed in other fields, it is relatively recent in our context. Results are presented on simulated and real data.

SPATIAL DEPENDENCE OF EXTREME METEOROLOGICAL VARIABLES
Robert Shooter, Lancaster University
A key consideration in spatial extremes is the form of the limiting model of the spatial process; namely whether it is asymptotically independent or asymptotically dependent. This has added importance in decision-making in meteorological contexts, since asymptotic dependence can be considered as a “conservative” estimate, leading to inefficient planning. An important quantity to consider in assessing this is the coefficient of tail dependence, \( \eta \). We look at estimating \( \eta \) by pooling data as opposed to using the existing pairwise method and how well each method estimates the known value of \( \eta \). Preliminary results from a stationary 1D Smith process suggest the true value of \( \eta = 1 \) is observed over a larger range than for the pairwise approach. Checks are made against theoretical conditional probabilities to further assess the performance of the pooled estimator against its pairwise counterpart.

DETERMINING THE DEPENDENCE STRUCTURE OF MULTIVARIATE EXTREMES
Emma Simpson, Lancaster University
Asymptotic independence and asymptotic dependence are important considerations when modelling multivariate extremes. There may be a complicated structure to this asymptotic behaviour, with only certain subsets of variables taking their largest or smallest values simultaneously, and this structure should be exploited when developing and fitting models for multivariate extremes. This poster discusses the method we are currently working on to determine the dependence structure of multivariate extremes. We consider variables in terms of their radial and angular components: a common technique in extreme value theory. In this case, the angular components lie in a unit simplex and, conditioning on the radial components being large, mass on the various sub-simplices corresponds to different subsets of variables being asymptotically dependent. Our current approach involves dividing the simplex into sections, and finding the conditional probability that a point lies in each section given that it is extreme in terms of the radial component. The poster outlines the benefits of this approach, and potential areas in which the method could be improved.
**Predicting influenza extreme epidemics**

**Maud Thomas, Université Pierre et Marie Curie**

Influenza viruses are responsible for annual epidemics, causing more than 500,000 deaths per year worldwide. A crucial question for resource planning in public health is to predict the morbidity burden of exceptional or extreme epidemics. A main goal of Extreme Value Theory (EVT) is to assess, from a series of observations, the probability of events that are more extreme than those previously recorded. Our objective is to predict the occurrence in the near future of exceptional or extreme influenza epidemics: to predict if in the next couple of weeks an unusual epidemic will occur. ILI (Influenza-like illness) has been shown to be a good proxy for influenza incidence. The ILI case definition consisted of a combination of fever above 39°C, myalgia, and respiratory symptoms. We collected the ILI incidence rates, that is the number of new cases per 100,000 individuals for week $t$, in France from 1984 to 2016 from the Sentinelles network: approximately 1,500 practising physicians in France voluntarily participate in disease surveillance through this system. The process described by the ILI incidence rates presents two different behaviours depending on whether it is in an epidemic phase or not. As a first step, we propose a two-state autoregressive Markov-switching model for this two-regime behaviour. Consider a sequence of pairs of random variables $(R_t, Y_t), t = 1, \ldots, n$ satisfying the following conditions

$$
P\{R_t | R_{t-1}, \ldots, R_1, Y_t, \ldots, Y_1\} = P\{R_t | R_{t-1}\}$$

$$
P\{Y_t | R_t, \ldots, R_1, Y_{t-1}, \ldots, Y_1\} = P\{Y_t | R_{t-1}, Y_{t-1}, Y_{t-2}\}.$$

- $(R_t)$ is hidden, that is, is not unobservable and assumed to follow a two-state discrete-time Markov chain with .
- $Y_t$ has a different behaviour given $R_t$

$$
Y_t = \phi^{(0)}(R_t) + \phi^{(0)}(Y_{t-1} - \phi^{(0)}(R_{t-1})) + \epsilon_t \text{ if } R_t = 0
$$

$$
Y_t = \phi^{(1)}(R_t) + \phi^{(1)}(Y_{t-1} - \phi^{(1)}(R_{t-1})) + \phi^{(1)}(Y_{t-2} - \phi^{(1)}(R_{t-2})) + \epsilon_t \text{ if } R_t = 1
$$

with $\phi^{(0)}, \phi^{(1)}_0, \phi^{(1)}_1, \phi^{(1)}_2 \in \mathbb{R}$ and $(\epsilon_t), t = 1, \ldots, n$ a sequence of i.i.d. random variables. Thus, we assume that, given $R_t = 0$, $Y_t$ behaves like an first order autoregressive process and, given $R_t = 1$, like a second order autoregressive process. $R_t$ represents the epidemic regime at week $t$ (0 codes for non-epidemic regime and 1 codes for epidemic regime), and $Y_t$ the ILI incidence rate at week $t$. As a second step, we define an epidemic to be extreme if at least one of the following two events occur: some $Y_t$ during the epidemic phase exceeds a (very high) threshold or the sum of the $Y_t$ within the epidemic exceeds a (slightly lower) level. We develop and compare prediction based on the autoregressive switching-Markov model and on the multivariate peaks over threshold models from (Rootzén et al, 2016).
Estimation of extreme quantiles in the common shaped tail model

Jasper Velthoen, Delft University of Technology

Let $Y$ denote the precipitation and $X$ the corresponding prediction by the numerical weather prediction model. These weather data do not follow a linear relationship due to the high uncertainties of the output of in the numerical weather prediction models. We propose the following common shaped tail model as the conditional quantile function of $Y$ given $X$: $Q_Y(\tau|X = x) = r(x) + e(\tau)$. We assume that $Y|X = x$ is in a max domain of attraction with a constant extreme value index that does not depend on $x$. We don’t assume any parametric structure on $r$. We develop a two-step procedure for estimating this model. First an estimation of $r$ is obtained using a non-parametric version of the composite quantile estimator proposed by Zou et.al. (2008). In the second step, methods from univariate extreme value theory are applied to the residuals to obtain the extreme quantile estimations. We have extensively studied the literature on conditional extreme quantile estimation, that incorporates covariates in the model. We compare our estimator with the Power-transformed linear quantile estimator from Wang et.al. (2013) and the Kernel estimator from Daouia et.al. (2013), in a simulation study and also in a real data set on the precipitation (provided by the Dutch Meteorological Institute, KNMI).

Asymptotic Independence of Bivariate Order Statistics

Florian Wisheckel, Universität Würzburg

It is well known that an extreme order statistic and a central order statistic as well as an intermediate order statistic and a central order statistic from a sample of i.i.d. univariate random variables get asymptotically independent as the sample size increases. This result can be extended to bivariate random variables, where the order statistic are taken componentwise.

On characteristic functions of products of two random variables

Jiang Xia, The University of Manchester

Many variables in the real world can be assumed to follow the normal distribution. That is, we can write $U = \mu + \sigma X$, where $X$ is a standard normal variable, $\mu$ is the mean and $\sigma$ is the standard deviation. But often the mean and standard deviation are themselves random variable, so $U$ involves a product of two random variables. Motivated by Schoenecker and Lugibuhl [IEEE Signal Processing Letters, 23, 2016, 644-647], we study the distribution of the product of two independent random variables, one of them being the standard normal random variable and the other allowed to follow one of nearly fifty distributions. We give explicit expressions for the characteristic function of the product. The mathematical derivations are verified by simulations.

Tail indices and Scale Parameters of Equity Return Distributions

Xiaolei Xie, University of Copenhagen

We consider an investor with preferences that accord with Generalized Disappointment Aversion. Such an investor cares about downside risk and we assume he recognizes the heavy tail feature of asset return distributions. We argue that when a market is dominated by rational investors of this kind, the return distributions of equities that are actively traded in this market must have very similar tail indices. On the other hand, whether or not all equities in a multivariate model have the same tail index is a dividing issue for multivariate GARCH models proposed in the literature. Therefore, it is important to analyze data of real equity returns and see how close to each other the tail indices actually are. In this work empirical results are also presented and they appear to support the conclusion that the tail indices are very similar, with respect to the confidence bands of estimation.
ON THE MAXIMUM AND MINIMUM FOR CLASSES OF BIVARIATE DISTRIBUTIONS

Yuanyuan Zhang, The University of Manchester

Given \((X,Y)\), the distributions of \(U = \min(X,Y)\) and \(V = \max(X,Y)\) are of interest in many areas. We derive the distributions of \(U\) and \(V\) for fifteen families of distributions of \((X,Y)\), including the bivariate normal and bivariate \(t\) distributions.
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